Exercise 1.2.9

a. $0 \cdot x = 0$

$x \cdot 1 = x + 0$ (Id+) (Id·)

$x \cdot (1 + 0) = x + 0$ (Id+)

$x \cdot 1 + x \cdot 0 = x + 0$ (D)

$x + x \cdot 0 = x + 0$ (Id·)

$\therefore x \cdot 0 = 0$ (cancellation)

$\therefore 0 \cdot x = 0$ (A·)

b. If $x \neq 0, y \neq 0$, then $xy \neq 0$, thus if $xy = 0$, then either $x = 0$ or $y = 0$.

If $x \neq 0, y \neq 0$, then $(x^{-1}y^{-1})(xy) = 1$, hence by (a), $xy \neq 0$.

Why can't $1 = 0$?

$x \cdot 1 = x$ (Id·)

$x \cdot 0 = 0$ (proposition)

This contradicts $1 = 0$

c. $(-x)y = -xy = x(-y)$

$x(y + (-y)) = xy + (-(xy))$ (Inv +)

$xy + x(-y) = xy + (-(xy))$ (D)

$\therefore x(-y) = -(xy)$ (cancellation)

d $(-x)(-y) = xy$

$(-x)(-y) = -(-x)(y)$ (proposition)

$= -(-(xy))$ (proposition)

$= xy$ (proposition)